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An approach to setting inflation and discount rates

DR HUGH MILLER AND TIM YIP

Dr Hugh Miller

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1 INTRODUCTION

Setting inflation and discount assumptions is a core part of many actuarial tasks. AASB 1023 requires that provisions for general insurance liabilities include an allowance for inflation and are discounted at the risk-free rate. However, there are a number of issues associated with setting these assumptions, including the following:

- What model should be adopted to fit the yield curve from observable risk-free securities?
- How should a discount rate curve be extrapolated beyond the last observable risk-free asset?
- What are appropriate long-term discount and inflation rates?
- How should inflation rate assumptions vary with respect to changes in risk-free rates?

This paper presents an approach to assumption setting that addresses these questions in a consistent and coherent manner. The approach is faithful to the observed behaviour of the market and previous research on the topic.

2 BACKGROUND

The approach presented in this paper relies heavily on previous research undertaken at consultancy firm Taylor Fry, as well as some other sources. The most important papers relied upon are described here. These papers in turn have more comprehensive lists of references for the interested reader.

2.1 Miller (2010)

This paper, "Towards a better inflation forecast", investigated inflation assumptions and the relationship between inflation and risk-free forward rates. The most important conclusions were the following:

- Available industry forecasts, such as those by Access Economics, had some use in predicting inflation in the short term, but limited effectiveness in medium- to long-term prediction.

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- There is an index of models that can describe the relationship between inflation and risk-free forward rates. These range from “fixed rate” models (the long-term inflation rate never changes) to “fixed gap” models (where a 1% increase in forward rates causes a 1% increase in inflation). They are indexed by the “inflation parameter” θ, with $\theta = 0$ corresponding to a fixed rate and $\theta = 1$ corresponding to a fixed gap.
- A range of tests showed that the inflation parameter is closer to 0 than 1. Estimates for the parameter using a range of approaches gave a range of 0–0.3 for the inflation parameter for average weekly earnings (AWE) inflation.
- There is reasonable historical evidence that AWE and labour price index (LPI) inflation are different across states. Higher rates for mining states (Western Australia and Queensland) appear justified, as are lower rates for some other states (New South Wales, Victoria and Tasmania).

2.2 Mulquiney and Miller (2014)

This paper “A topic of interest – how to extrapolate the yield curve in Australia”, contained a detailed look at yield-curve extrapolation, drawing from data in Australia and overseas. Relevant findings include the following:

- Medium- to long-term forward rates (around 10 years) have only partial ability to predict very long-term rates (30 years and beyond). This is indicative of long-term reversion of the forward rate.
- The long-term forward rate can be thought of as the combination of inflation expectations, real interest rate expectations, a risk premium and convexity adjustment. A long-term forward rate assumption in the range 5.4% to 6.2% was judged to be reasonable at this time.
- A linear reversion shape to the long-term forward rate was judged reasonable, although other shapes are possible.
- The rate of reversion was observed to be slow, based on several different tests. Reversion to the long-term forward rate somewhere between 40 and 80 years was judged reasonable.

2.3 Intergenerational reports

The Australian Treasury regularly publishes the *Intergenerational Report*, which contains long-range projections of the Australian economy. The most recent was published in 2015, and included the following assumptions:

- long-term bond rates of 6.0%
- long-term CPI inflation of 2.5%
- long-term AWE inflation of 4.0%.

These assumptions are consistent with previous reports.

2.4 Other background information

A number of changes have been observed in Australian bond markets in recent years that have had a large impact on discount rates and how they are forecast. First, the last few years have seen very low bond rates (see Figure 1). A consequence of this is that discount rate forecasts have become more sensitive to the assumptions adopted in relation to mean reversion, as the 10-year bond rate is no longer close to the long-term bond rate.

Second, the number and term of Australian government bonds on issue have increased. In June 2005, 11 bonds were on issue with maximum term of 12 years. In March 2014, 21 bonds were on issue with the longest term, 22 years. This increases the possible complexity of the yield curve shape and decreases the scope for a fast reversion of yields.

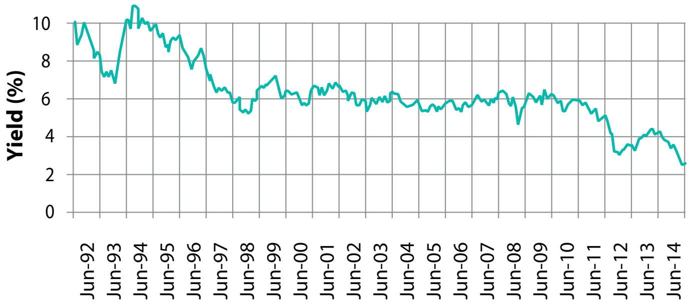

Figure 1: 10-year forward rate, Australian government bonds, 1992–2014.

Source: Reserve Bank of Australia, statistics series F16, available at [http://www.rba.gov.au/statistics/tables/xls-hist/f16hist.xls; http://www.rba.gov.au/statistics/tables/xls-hist/f16hist-2009-2015.xls].

3 SETTING DISCOUNT RATES

3.1 Objectives of yield curve fitting

The main objectives of yield curve fitting are to obtain a set of forward rates that:

- **is smooth:** This is generally viewed as a desirable feature. Additionally, non-smooth yield curves tend to present more arbitrage opportunity, so they should be less frequent in practice.
- **fits observable bond prices well:** Each bond is viewed as the sum of zero coupon bonds. A good fit means that the price of those cash flows based on the forward rates is close to the observed bond price.
- **exhibits reversion over the long term:** The model should be able to impose reversion to the long-term rate at terms beyond observable bond prices.

3.2 Adopted approach – constrained cubic spline model

To achieve the objectives outlined in Section 3.1, we have assumed that forward rates follow the shape illustrated in Figure 2. The model assumes a cubic spline shape between term 0 and term t_3 with two additional interior knots t_1 and t_2. Further it assumes linear reversion between t_3 and t_4, with a constant forward rate beyond t_4.

In terms of equations, the model illustrated in Figure 2 is is expressed as:

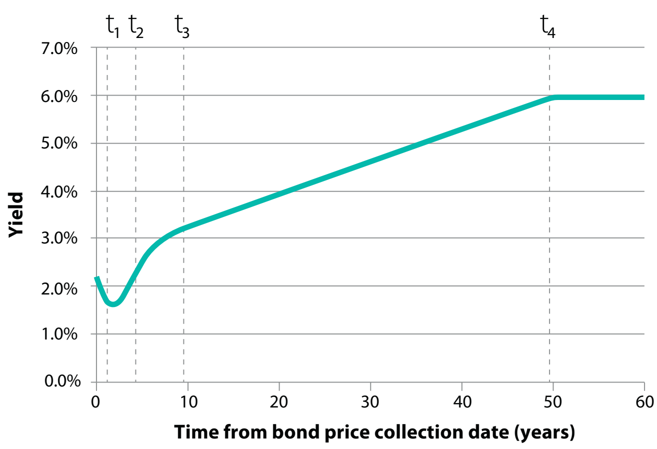

Figure 2: Schematic for adopted forward rate fitting shape.

$$f(x) = a + bx + d\lfloor x - 0 \rfloor^3 + e\lfloor x - t_1 \rfloor^3 + f\lfloor x - t_2 \rfloor^3 + g\lfloor x - t_3 \rfloor^3 \quad (1)$$

Here $\lfloor x \rfloor = x$ when $x > 0$ and $\lfloor x \rfloor = 0$ otherwise. Additionally, we impose the following constraints on the curve:

1. Reversion to the long- term rate f^* at term t_4:

   $$f(t_4) = a + bt_4 + d(t_4 - 0)^3 + e\lfloor t_4 - t_1 \rfloor^3 + f(t_4 - t_2)^3 + g(t_4 - t_3)^3 = f^* \quad (2a)$$

   For this particular constraint we have set $f^* = 6.0\%$ and $t_4 = 50$.

2. Linear reversion between terms t_3 and t_4. So in this region, $f''(x) = 0$. Spitting constant and x components gives:

   $$d + e + f + g = 0 \text{ and} \quad (2b)$$

   $$et_1 + ft_2 + gt_3 = 0 \quad (2c)$$

The equations (2a)-(), (2b) and (2c) can be solved simultaneously to eliminate e, f and g from (1), giving:

$$e = \frac{\{f^* + a + bt_4 + dt_4{}^3\} + d\{-t_3(t_4 - t_2)^3/(t_3 - t_2) + t_2(t_4 - t_3)^3/(t_3 - t_2)\}}{-(t_4 - t_1)^3 + (t_3 - t_1)(t_4 - t_2)^3/(t_3 - t_2) - (t_2 - t_1)(t_4 - t_3)^3/(t_3 - t_2)} \quad (2d)$$

$$f = -\frac{e(t_3 - t_1) + d(t_3 - 0)}{t_3 - t_2} \quad (2e)$$

$$g = -(d + e + f) \quad (2f)$$

with the remaining parameters estimated using observed bond prices. While equations (2a), (2b) and (2c) could be used to eliminate any three parameters, we have found eliminating the last three to produce the more stable numerical results when fitting to observed prices.

If B_j is the observed price of the jth bond, and $\hat{B}_j$ is the corresponding price estimate using the forward rate curve, then the parameters in equations (2d), (2e) and (2f) are chosen to minimise the weighted squared error:

$$\text{Error} = \sum_j w_j \left(B_j - \hat{B}_j\right)^2 \quad (3)$$

where the weight of each bond w_j is equal to $1/D_j^2$ with D_j the modified duration of bond j.

Two of the parameters in equation (2d), t_4 and f^*, are set subjectively as described in section 3.3. The remaining unknown parameters a, b and d and knots t_1, t_2 and t_4 are chosen to minimise (3) using a non-linear optimiser. We have implemented this using the Solver functionality in Microsoft Excel.

3.3 Further comment on subjective assumptions

There are two important assumptions in this fitting model that are required to be set subjectively. These are the choices for t_4, the point at which the ultimate long-term rate is achieved (here it is a term of 50 years), and the long-term rate itself f^* set to 6.0%. These assumptions have been selected with reference to the studies cited in section 2.

3.4 Alternative approaches for yield curve fitting

Before adopting the above cubic spline–based fitting approach, we considered the approaches detailed in Nelson and Siegel (1987), Svensson (1994), Li, DeWetering, Lucas, Brenner and Shapiro (2001), and Smith and Wilson (2001).

While all could probably be amended to meet the objectives set out in section 3.1, none did so "out-of-the-box". Further, differences in fitting approaches tend to be immaterial apart from the assumptions related to extrapolation: as long as the curve is sufficiently flexible, it should give a reasonable fit of the observable securities. Other comparisons of approaches exist – see for instance Bolder and Gusba (2002).

4 SETTING INFLATION RATES

4.1 Our approach

Our approach to forecasting inflation is as follows:

1. Adopt a third party econometric forecast in the short term (the first two years).
2. For the fifth year and beyond, adopt an inflation rate based on the estimated forward rate:

$$i(t) = i^* + \theta\{f(t) - f^*\} \quad (4)$$

3. For the third and fourth years, linearly blend between the two approaches.

This approach is an extension of the model proposed in Miller (2010), which used an equation similar to (4) to estimate medium-term inflation expectations as a function of long-term bond yields. The formulation in (4) makes the further strong assumption that this relationship holds over the term of the yield curve, so inflation forecasts mean-revert with a similar shape (but smaller amplitude) in line with forward rates. While mean-reversion of inflation rates is intuitively appealing, we have not formally tested the speed of reversion relative to that of bond yields. Other approaches to inflation reversion are certainly possible; the attraction of (4) is that linking the inflation and yield curves to have similar shapes makes liability movements more predictable over time.

The blending in the third and fourth years helps avoid a cliff in forecasts, should the econometric and formula based forecasts materially differ.

In terms of explicit assumptions:

- We have selected $i^* = 2.5\%$ for CPI inflation (the centre of the RBA target band and consistent with the 2015 Intergenerational Report), $i^* = 3.6\%$ for LPI inflation (consistent with long-run historical averages) and $i^* = 4.0\%$ for AWE inflation (consistent with the Intergenerational Report and long-run averages).
- We have selected $\theta = 0.5$ as the inflation parameter. Although higher than estimates in Miller (2010), it captures some of the sensitivity of inflation to nominal interest rates and provides a balance between the "fixed inflation" and "fixed gap" extremes.
- We apply capping to the CPI forecast so that it does not exit the RBA target band (2.0%–3.0%). That is, for CPI the adopted formula is slightly modified:

$$i(t) = \min(3.0\%, max(2.0\%, i^* + \theta\{f(t) - f^*\}))$$

- $f(t)$ and f^* are consistent with the previous section, with $f^* = 6.0\%$ (consistent with the *Intergenerational Report*).

4.2 Modifiers for difference states

In addition to the Australia-level forecasts in the previous subsection, we add modifiers to certain states:

- + 0.5% for LPI and AWE inflation for Western Australia and Queensland.
- – 0.25% for LPI and AWE inflation for New South Wales, Victoria and Tasmania.

Although these differentials were estimated in 2010, they have proven reasonably accurate over the past few years: see Table 1. However, these factors will have to be reviewed regularly; the cyclical trends in resource markets will tend to influence appropriate choices for state-based differences, and there is already some early evidence of Western Australian inflation falling back to national levels (see for example Nicholls & Rosewall 2015)

Table 1: Historical AWE growth differentials for each state.

Time period	State difference from national average								Aust
	NSW	VIC	QLD	SA	WA	TAS	NT	ACT	AWE
Jun 02 – Jun 06	–0.3%	–0.4%	0.4%	0.3%	1.3%	–1.3%	0.6%	1.2%	4.5%
Jun 06 – Jun 10	–0.7%	–0.5%	0.9%	–0.9%	1.8%	0.6%	–0.2%	0.4%	4.9%
Jun 10 – Jun 14	–0.6%	–0.4%	0.3%	–0.5%	1.5%	0.8%	1.5%	0.6%	3.9%

Note: Based on ABS average weekly full time earnings, trend series, available at:.
Source: [http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/6302.0May%202012?OpenDocument; http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/6302.0Nov%202016?OpenDocument].

5 CONCLUSION

This paper presents a combined approach to inflation and discount rate assumption–setting that should be appropriate for a wide range of actuarial contexts. Interested readers are encouraged to seek out the referenced papers, as well as contact Taylor Fry directly for further information.

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